Electromagnetism

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1 Electrostatics

Electrostatics is the special case of electromagnetism where all charges are at rest. The theory is essentially based on two empirical laws.

Theorem 1.1: Superposition principle

The total force on a charge is the vector sum of the forces exerted by all other charges.

This theorem also holds in electrodynamics.

Theorem 1.2: Coulomb's law

The force $\mathbf{F}$ between two point charges q_1 and q_2 separated by a distance r is given by

$$\mathbf{F} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}\hat{r}.$$

Here, ε_0 is the permittivity of free space, and $\hat{r}$ is the unit vector pointing from one charge to the other.

This definition is a bit awkward, since it only defines the force between two point charges. To get a more general definition, we introduce the electric field

Definition 1.3: Electric Field

The electric field $\mathbf{E}$ at a point in space is defined as the force $\mathbf{F}$ experienced by a test charge q placed at that point, divided by the magnitude of the test charge:

$$\mathbf{E} = \frac{\mathbf{F}}{q}.$$

On a side note, the test charge should be infinitesimally small, so that it does not disturb the existing electric field.

For an arbitrary charge distribution, the electric field can be calculated using the superposition principle and Coulomb's law.

$$\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0}\int \frac{\rho(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|^2}\hat{r}\,dV'.$$

The problem is that this integral is a vector integral, which is annoying to work with. To get around this problem, we introduce the electric potential.

Definition 1.4: Electric Potential

The electric potential V at a point in space is defined as the work done by an external agent in bringing a unit positive test charge from infinity to that point, without any acceleration.

$$V(\mathbf{r}) = -\int_\infty^{\mathbf{r}} \mathbf{E}\cdot d\mathbf{l}.$$

For a charge distribution, the electric potential can be calculated using the superposition principle and Coulomb's law.

$$V(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0}\int \frac{\rho(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|}\,dV'.$$

This is a scalar integral, which is much easier to work with than the vector integral for the electric field.

To compute the electric field in symmetric cases, we can use Gauss's law, often just to avoid memorizing the electric field of a sphere, a line, or a plane.

Theorem 1.5: Gauss's Law

The electric flux Φ_E through a closed surface S is given by

$$\Phi_E = \oiint_{\partial S} \mathbf{E}\cdot d\mathbf{A} = \frac{Q_{\text{enc}}}{\varepsilon_0}.$$

Exercise 1.6: Sphere

Calculate the electric field of a uniformly charged sphere with total charge Q and radius R using Gauss's law.

Ex 1.6

Knowing the three standard cases (sphere, line, plane) is very useful, since they can be used as building blocks for more complicated charge distributions due to superposition.

Exercise 1.7: Line

Calculate the electric field of an infinitely long line of charge with linear charge density λ using Gauss's law.

Exercise 1.8: Plane

Calculate the electric field of an infinite plane of charge with surface charge density σ using Gauss's law.

A less standard example but short and fun is the following:

Exercise 1.9: Cube

Given a cube of side length a with a charge Q placed at one of its corners, compute the electric flux through one of the far faces of the cube.

Ex 1.9

1.1 Spieglein Spieglein an der Wand

The method of images is a powerful technique for solving electrostatic problems with certain symmetries that has previously appeared in EuPhO problems. Consider the following problem:

Example 1.10: Image Charge

A point charge q is located at a distance d above an infinite grounded conducting plane. Calculate the electric field and potential in the region above the plane.

Before solving this, a bit of theory. The potential V in a region of space with given charge distribution uniquely determines the electric field $\mathbf{E}$, given certain boundary conditions. Problems with equal boundary conditions will have the same solution, even if the charge distribution outside the region of interest is different.

Solution. We can solve the problem by replacing the conducting plane with an image charge of $-q$ located at a distance d below the plane. This configuration has the same boundary conditions as the original problem, since the potential on the plane is zero in both cases. Therefore, the electric field and potential in the region above the plane are given by the superposition of the fields and potentials of the real charge q and the image charge $-q$.

This method can also be used in magnetostatics. If interested, look at EuPhO 2017 P3.

1.2 Electric Dipole

As we will see, in magnetism the simplest object is a magnetic dipole, which also exists in electrostatics as an electric dipole. It is more intuitive in my opinion which is why I will introduce it here.

An electric dipole consists of two equal and opposite charges separated by a small distance.

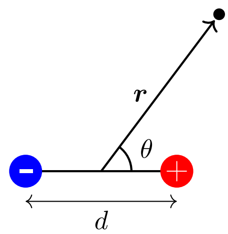


Figure 1: A physical electric dipole consisting of two point charges

Some important quantities related to the electric dipole are:

- The dipole moment $\mathbf{p}$, defined as $\mathbf{p} = q\mathbf{d}$, where $\mathbf{d}$ is the vector pointing from the negative charge to the positive charge.
- The electric field $\mathbf{E}$ at a point $\mathbf{r}$ in space due to the dipole, which can be calculated using the superposition principle and Coulomb's law.

When we place an electric dipole in a uniform electric field $\mathbf{E}$, it experiences a torque τ given by

$$\tau = \mathbf{p} \times \mathbf{E}.$$

This torque tends to align the dipole moment $\mathbf{p}$ with the electric field $\mathbf{E}$. The potential energy U of the dipole in the electric field is given by

$$U = -\mathbf{p} \cdot \mathbf{E}.$$

If you have a non-uniform electric field, the way to calculate the force on the dipole is to calculate the force on each charge and then add them up.

2 Magnetostatics

Magnetostatics is the special case of electromagnetism where all currents are steady (not changing with time). The theory is essentially based on two empirical laws. The first one is yet again the superposition principle.

Theorem 2.1: Biot-Savart Law

The magnetic field $\mathbf{B}$ at a point in space due to a steady current distribution is given by

$$\mathbf{B}(\mathbf{r}) = \frac{\mu_0 I}{4\pi} \int \frac{d\mathbf{l} \times \hat{r}}{r^2}.$$

Here, μ_0 is the permeability of free space.

Similarly to Coulomb's law this definition is a bit awkward, due to the vector integral. Similarly to Gauss's law in electrostatics, we can use Ampère's law to compute the magnetic field in symmetric cases.

Theorem 2.2: Ampère's Law

The line integral of the magnetic field $\mathbf{B}$ around a closed loop C is given by

$$\oint_C \mathbf{B} \cdot d\mathbf{l} = \mu_0 I_{\text{enc}}.$$

Exercise 2.3: Straight Wire

Calculate the magnetic field at a distance r from an infinitely long straight wire carrying a current I using Ampère's law.

Ex 2.3

Exercise 2.4: Solenoid

Calculate the magnetic field inside an ideal solenoid with n turns per unit length carrying a current I using Ampère's law.

Other symmetries exist but are pretty rare.

To compute the force of a magnetic field on a charge distribution, we can use the Lorentz force law.

Theorem 2.5: Lorentz Force Law

The force $\mathbf{F}$ on a charge q moving with velocity $\mathbf{v}$ in a magnetic field $\mathbf{B}$ and an electric field $\mathbf{E}$ is given by

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B}).$$

2.1 Magnetic Dipole

The simplest object in magnetism is the magnetic dipole, which can be thought of as a tiny current loop.

Definition 2.6: Magnetic Dipole Moment

The magnetic dipole moment $\mathbf{m}$ of a current loop is defined as

$$\mathbf{m} = I\mathbf{A}.$$

Here, I is the current and $\mathbf{A}$ is the area vector of the loop, which has a magnitude equal to the area of the loop and a direction given by the right-hand rule.

The magnetic field $\mathbf{B}$ at a point $\mathbf{r}$ in space due to a magnetic dipole can be calculated using the Biot-Savart law. It is given by[1]

$$\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi} \left(\frac{3(\mathbf{m} \cdot \hat{r})\hat{r} - \mathbf{m}}{r^3} \right).$$

Similarly to the electric dipole, when we place a magnetic dipole in a uniform magnetic field $\mathbf{B}$, it experiences a torque τ given by

$$\tau = \mathbf{m} \times \mathbf{B}.$$

This torque tends to align the magnetic dipole moment $\mathbf{m}$ with the magnetic field $\mathbf{B}$. The potential energy U of the magnetic dipole in the magnetic field is given by

$$U = -\mathbf{m} \cdot \mathbf{B}.$$

Exercise 2.7: EuPhO 2019 P2

A solid, homogeneous spherical ball of mass m and radius R is made of insulating material and has charge Q distributed uniformly throughout its volume. The ball is placed on a large horizontal surface, and set in rolling motion without slipping in such a way that its center starts to move with initial horizontal velocity v_0. There is a uniform magnetic field (flux density) of magnitude B perpendicular to the surface. The coefficient of static friction is large enough to prevent the ball from slipping on the surface. The moment of inertia of the ball about an axis through its center is $2mR^2/5$. Describe the motion of the center of the ball and the shape of its trajectory

3 Induction

Induction is the phenomenon where a changing magnetic field induces an electric field. The theory is based on Faraday's law of induction.

[1] This formula should be memorized, since it is a bit tricky to derive.

Theorem 3.1: Faraday's Law of Induction

The electromotive force (EMF) $\mathcal{E}$ induced in a closed loop C is equal to the negative rate of change of the magnetic flux Φ_B through the loop:

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}\iint_S \mathbf{B}\cdot d\mathbf{A}.$$

In principle with EMF we will usually just take it as a voltage, which is the potential difference between two points. The minus sign in Faraday's law is a consequence of Lenz's law, which states that the induced EMF will always act to oppose the change in magnetic flux that caused it. This is a consequence of energy conservation.

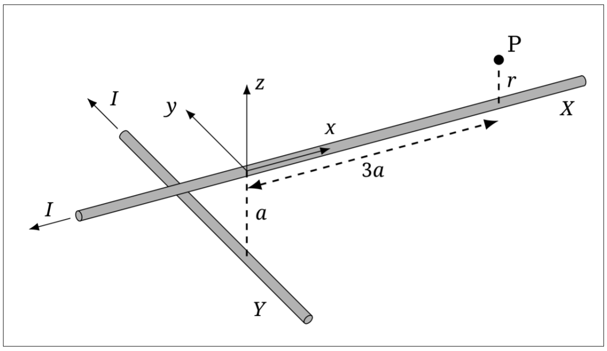
Exercise 3.2: Thomson's Jumping Ring

A conducting ring is placed around the vertical core of an electromagnet. When the current in the electromagnet is switched on, the ring jumps up. Explain this phenomenon using Faraday's law of induction and Lenz's law.

4 My Favorite Problem

Example 4.1: EuPhO 2025 P3, part b)

Consider two infinite, straight, thin wires (wires X and Y), each carrying a current I as shown in the figure. The x-axis coincides with wire X, while wire Y is parallel to the y-axis and passes through the point $(0,\,0,\,-a)$. Let P be the point $(3a,\,0,\,r)$. Assuming $r \ll a$, calculate d, the distance of closest approach of the magnetic field line that passes through P to the wire X.



This problem is very hard, but technically can be solved with your high school physics knowledge. The solution is very neat and elegant, using a very small amount of actual computations. Have fun.