Mathematical Methods

Problem Set

The following problems are designed to help you practice the key methods covered in the lecture. To master the techniques presented it is nevertheless essential to do practice beyond this sheet, and to apply the methods to problems in other contexts. You for sure can make use of them in some other lectures this week.

Problem 1 Linearisation

Linearise the following functions around the specified points:

a) $f(x) = \sin(x)$ around $x_0 = \pi/4$

b) $f(x) = \ln(x)$ around $x_0 = 1$

c) $f(x) = e^x$ around $x_0 = 3$

d) $f(x) = \sqrt{x}$ around $x_0 = 4$

e) $f(x) = \sin(\ln(x))$ around $x_0 = 1$

f) $f(x) = \cos(x^3 - 2x) + \sqrt{x^2 + 1}$ around $x_0 = 5$

Problem 2 Numerical Differentiation

Given is the function $f(x) = x^3 - x - 1$. Find the real root of this polynomial using both the bisection method and Newton's method.

Problem 3 Integration

Find the area under the curve of $f(x) = x^2$ from $x = 0$ to $x = 3$ using integration and the trapezoidal rule. Show that as the number of trapezoids increases, the approximation converges to the exact value.

Problem 4 Statistics

A dataset consists of the following values: $2, 4, 4, 4, 5, 5, 7, 9$. Calculate the mean, median, mode and standard deviation of the dataset. Furthermore, draw a boxplot to visualize the distribution of the data.

Problem 5 Error Propagation

Consider the function $f(x, y) = x^2 + y^2$. If $x = 3.0 \pm 0.1$ and $y = 4.0 \pm 0.2$, calculate the propagated uncertainty in f using the error propagation formula.